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We suggest a new type of radiation (acoustically induced radiation – AIR), which is generated by ultrarelativistic particles channeling in crystal along a crystal plane (or axis), which is bent by a transverse acoustic wave. The AIR mechanism allows to make an undulator with characteristics inaccessible in the undulators based on the motion of particles in the periodic magnetic fields and also in the field of the laser radiation. The intensity of AIR can easily be made larger than the intensity of the radiation in a linear crystal and can be varied in a wide range by varying the frequency and the amplitude of the acoustic wave.

41.60, 61.85+p

We suggest the new type of the undulator radiation (acoustically induced radiation – AIR), which is generated by the ultrarelativistic charged particle channeled in a crystal along a crystal axis or a crystal plane, which is bent by a transverse acoustic wave (AW). In this letter we consider channeling of a positron along the plane bent by the transverse monochromatic standing AW of the frequency within $1 - 10^2$ MHz range. Due to bending of the crystal planes caused by the AW, the beam of channeled positrons penetrating through the channel oscillates in the transverse direction as illustrated in figure 1. Transverse oscillations of positrons caused by the AW become the effective source of radiation of undulator type due to the constructive interference of the photons emitted from the similar parts of the trajectory. As we demonstrate below, the number of oscillations can vary in a wide range from few up to few hundreds oscillations per *cm* depending on the chosen parameters.

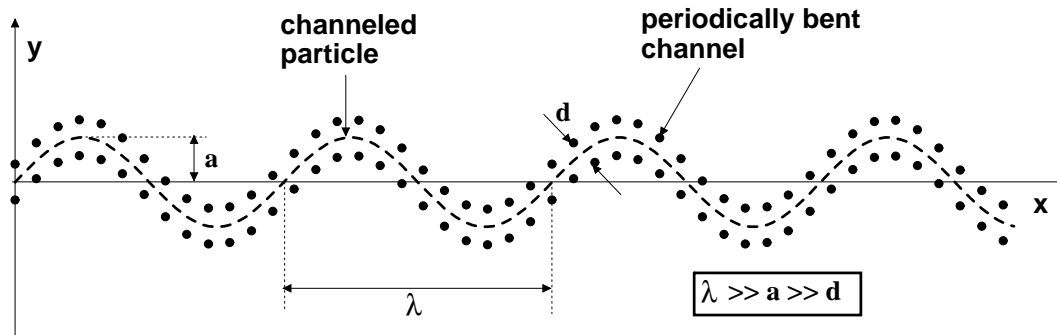


FIG. 1. Beam of particles channeled through the crystal bent by the transverse acoustic wave.

In addition to AIR, channeled positrons emit ordinary channeling radiation [1] due to their transverse motion inside the channel. However, the frequency of these oscillations is much higher than the frequency of the transverse oscillations caused by the AW (see the estimates below). Therefore, two motions are well separated, and the AIR mechanism can be treated independently from the ordinary channeling radiation. A similar situation occurs in a one-arc bent crystal, where channeled charged particles generate additional synchrotron type radiation due to the curvature of the channel [2,8]. This component of the total radiation intensity leads to the undulator effect in the channel periodically bent by AW. We demonstrate that the intensity of AIR can be made larger than the ordinary channeling radiation. The important feature of the AIR consists in the possibility to vary significantly the intensity and the shape of the spectral distribution of AIR by varying the frequency and the amplitude of AW. The suggested mechanism of AIR allows to make an undulator with the parameters N and p varying in a wide range (where N is

the number of periods of the undulator and p is the undulator constant) which is inaccessible in the undulators based on the motion of charged particles in the periodic magnetic fields and also in the field of the laser radiation [3]. In the suggested scheme, AIR is generated by the relativistic charged particles, with relativistic factors $\gamma = \varepsilon/mc^2$ in the range $\gamma = 1 - 10^7$ and above (c is the velocity of light, m is the mass of the particle). The large range of γ available in the modern colliders at present or in nearest future for various charged particles, both light and heavy, together with the wide range of frequencies and the amplitudes possible for AW in crystals allows to generate the AIR photons with the energies up to the TeV region.

The phenomenon, which we consider in the present paper, is an example of the charged particle channeling in a bent crystal. The process of channeling of the charged particles in a bent crystal is presently of great current interest (see e.g. [4,5]), because of its possible practical applications for the manipulation of charged particle beams of high energy. In these papers, both theoretical and experimental, bent crystals were assumed or used as specially prepared or grown. Here we suggest completely different way of bending the crystal based on the use of the acoustic waves. It is important to note that the condition for channeling in a bent crystal (see e.g. [2,5]) can be fulfilled in this case too. As a result, motion of particle in the field of the channel bent by the transverse AW becomes periodic and thus acquires essential features of the motion of the particle in the periodic magnetic field. However, parameters of particle trajectory, such as the amplitude or the oscillation period, are not accessible in magnetic fields available at present in laboratory conditions.

Now let us analyze the conditions which must be fulfilled when considering the bent crystal as an undulator. Consider an ultra-relativistic particle (of mass m , charge $q > 0$, energy ε , relativistic factor $\gamma \gg 1$, $v \approx c$) channeling near a crystallographic plane. Extension to the case of axial channeling of electrons is rather straightforward. Let the transverse monochromatic AW be transmitted along the direction of projectile's motion, and let v_t , ν , $\lambda = v_t/\nu$, a be the AW phase velocity, the frequency, the wave length and the wave amplitude, respectively. Under the action of AW the planar channel, linear initially, will be bent.

Bending of a channel by the AW becomes significant if the AW amplitude becomes larger than the interplanar distance, d , $d \ll a$. In this letter we analyze the case when the shape of the channel does not vary much during the time of flight, $\tau = L/c$, of the particle through the crystal of the thickness L , i.e. τ is much smaller than the AW period T : $\tau \ll T$. The channeling process in a bent crystal takes place if the maximal centrifugal force in the channel, $m\gamma v^2/R_{min}$ (where R_{min} is a minimum curvature radius of the bent channel) is less the force caused by the maximal interplanar field U'_{max} [2,5]:

$$m\gamma v^2/R_{min} < qU'_{max}. \quad (1)$$

Provided (1) is fulfilled, the projectile, which enters the crystal under the angle θ much less than the critical angle θ_L , will move, being trapped in the channel, along the trajectory, which represents the instant shape of the accoustically bent channel (see figure 1):

$$y(x) = a \sin\left(2\pi \frac{x}{\lambda}\right), \quad x = [0 \dots L] \quad (2)$$

The frequency of the transverse oscillatory motion of a positron moving along the trajectory (2), $2\pi c/\lambda$, is much smaller than the frequency of the oscillatory motion inside the channel, $2\pi\sqrt{2U_{max}/m\gamma}/d$ [1], provided condition (1) is fulfilled and if $a \gg d$. The frequencies of the two types of motions are significantly different so that they can be treated independently.

The minimum curvature radius of the trajectory (2) is equal to $R_{min} = (\lambda/2\pi)^2/a$. Thus, decreasing λ and increasing a , we decrease R_{min} and increase the maximum acceleration of the particle in the channel. As a result, photon emission due to the projectile's acceleration in the bent channel may be significantly enhanced. Below we demonstrate that this radiation, which is emitted coherently from similar parts of the trajectory, may dominate considerably over the radiation caused by the acceleration of the particle in the linear channel.

For the trajectory (2) the relation (1) reads as

$$\nu^2 a < C \equiv \gamma^{-1} \cdot \left(\frac{v_t}{2\pi}\right)^2 \cdot \left(\frac{qU'_{max}}{mc^2}\right) \quad (3)$$

and determines ranges of ν , a and γ for which the channeling process as well as the undulator radiation, can occur for given crystal and crystallographic plane (the parameters U'_{max} and v_t are subject to the choice of a particular crystal and a plane) and for given projectile type, characterized by a rest mass and a charge.

Both the motion of the projectile in the bent channel and the spectrum of the generated electromagnetic radiation are of the undulator-type, only if $\lambda \ll L$, i.e. if the channeling particle oscillates many times within the length L of the crystal. The suggested undulator, 'AW + channeling particle', is characterized as any other undulator [3] by the frequency, $\omega_0 = 2\pi c/\lambda$, and the undulator parameter, $p = 2\pi\gamma a/\lambda$.

Figs. 2–3 illustrate the ranges of ν and a in which the channeling process for a positron and a proton in a carbon crystal is possible. The cases 2 and 3 correspond to the energies 50 and 500 GeV. The solid thick line in both figures represents the boundary $\nu^2 a = C$ (see (3)). In each figure the range of validity of (3) lies below this line. Dotted and dashed-dotted lines indicate the constant values of the undulator parameter p for a positron and a proton. The dashed lines correspond the constant values of the number of the AW periods per 1 cm: $N(\text{cm}^{-1}) = \lambda^{-1}$. All data presented in Figs. 2–3 and subsequently refer to a projectile channeling near the (110) crystallographic plane. We use v_t (10^5 cm/s): 11.64, 2.81 [6] and U'_{max} (GeV/cm): 12.0, 43.0 for C and W crystals, respectively [5].

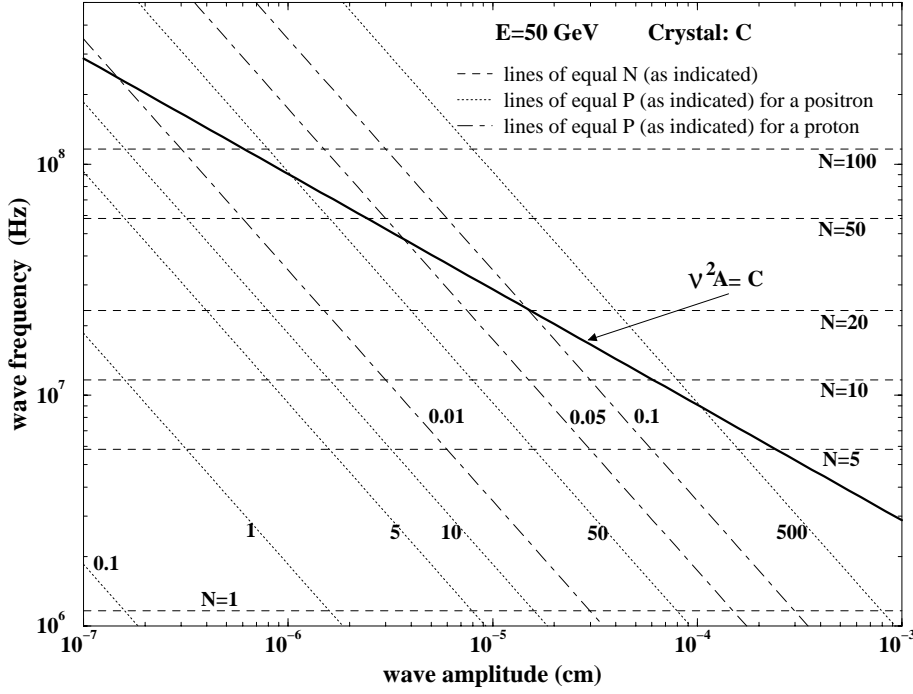


FIG. 2. The ranges of ν (in Hz) and a (in cm) in which the channeling process is possible for 50 GeV positron and proton in a carbon crystal. See also explanations in the text.

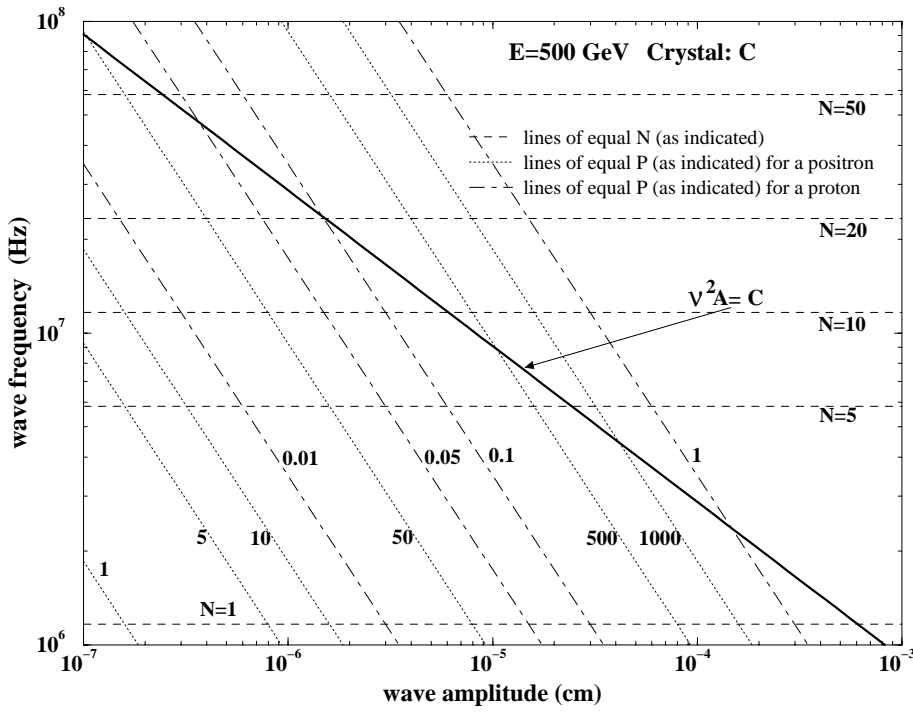


FIG. 3. Same as for Fig. 2 but for 500 GeV projectiles

Figs. 2–3 demonstrate that the parameters p and N vary in wide ranges: $N = 1 \dots 100$, $p = 0.1 - 500$ for projectile positron and $p = 0.001 - 0.1$ for a proton. These values are by more than an order of magnitude larger than those accessible in the undulators based on the motion of the charged particles either in periodic magnetic fields or in the field of the laser radiation [3].

In the limit $N, p \gg 1$ one can calculate the spectral intensity of radiation (per 1 cm) emitted by a projectile positron moving along the path (2), by utilizing the following formula deduced from general expression, given in [7], which has been obtained within the framework of quasi-classical approximation:

$$\frac{d\varepsilon}{d(\hbar\omega) L} = \alpha N p \frac{\omega'}{\omega} \left(G_1(y) + \left[1 + \frac{u^2}{2(1+u)} \right] G_2(y) \right) \quad (4)$$

where α is the fine structure constant, ω is the photon frequency, $\omega' = \omega(1+u)$, and the parameter $u = \hbar\omega/(\varepsilon - \hbar\omega)$ takes into account the correction due to the radiative recoil. The parameter y is defined as $y = (\omega'/\omega_0 \gamma^2 p)^{2/3}$ with $\omega_0 = 2\pi c/\lambda$. The functions $G_{1,2}(y)$ are

$$\begin{aligned} G_1(y) &= -2y^{5/2} \int_1^\infty dx \left[\pi - \arccos \left(1 - \frac{2}{x^3} \right) \right] \text{Ai}(yx) \\ G_2(y) &= -8y^{1/2} \int_0^\infty \frac{d\xi}{(\text{ch}\xi)^{5/3}} \text{Ai}'(y(\text{ch}\xi)^{2/3}) \end{aligned} \quad (5)$$

where $\text{Ai}(z)$, $\text{Ai}'(z)$ are the Airy function and its derivative respectively.

Using (4)–(5), we have calculated the spectral distributions of AIR at different parameters. The spectral distributions (per *cm*) of the radiation emitted by a 50 and 500 *GeV* positron moving along the trajectory (2) in carbon and tungsten crystals are plotted for various values of the undulator parameter p and the amplitude a in Figs. 4–5. These figures demonstrate that for a fixed frequency ν the intensity of radiation can be varied in a wide range by altering the AW amplitude, which is proportional to the value of the undulator parameter p . These figures illustrate as well the dependence of the spectral distributions on the energy of the particle. Comparison of the spectra presented in Figs. 4–5 with those in the case of channeling in the corresponding linear crystal [1] shows that the intensity of AIR can be made much larger than the corresponding intensity of radiation in the linear crystal case by choosing the appropriate AW parameters.

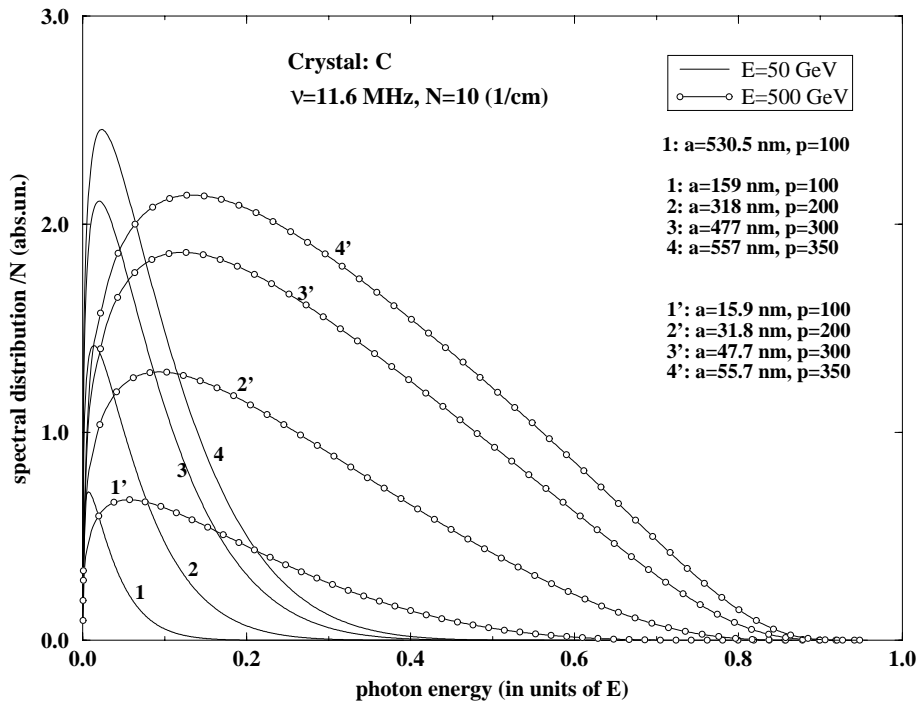


FIG. 4. The spectral intensity (per one period) of the AIR emitted by a 50 and 500 *GeV* positron in a carbon crystal calculated for the fixed AW frequency (as indicated) and for various parameters *a* and *p* (as indicated). See also explanations in the text.

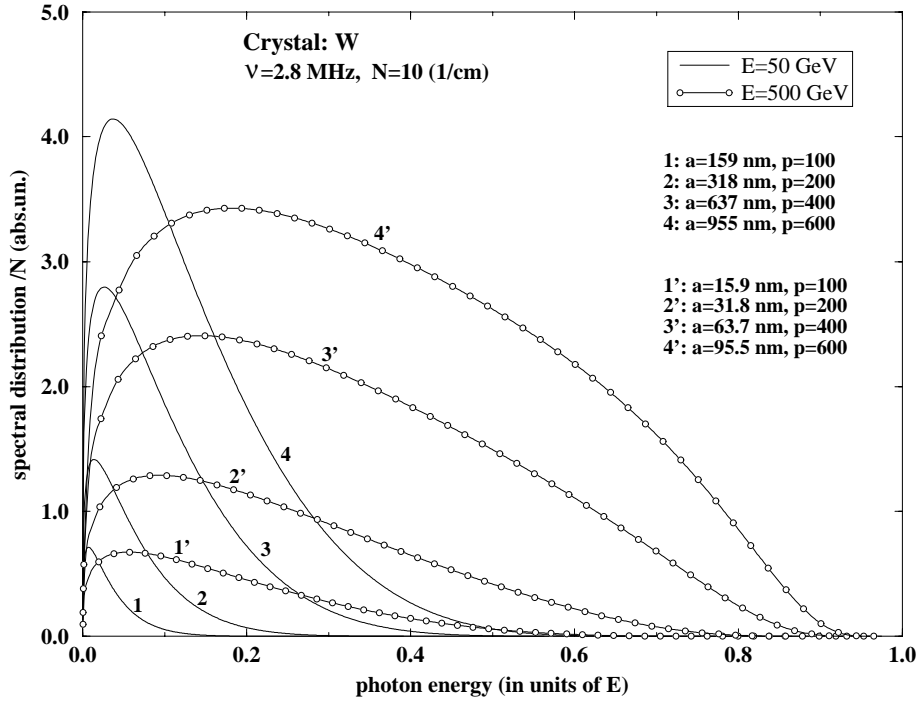


FIG. 5. Same as for Fig. 4 but for a tungsten crystal.

The spectral intensities (per 1 *cm*) of the radiation emitted by a 50 *GeV* positron moving along the trajectory (2)

are compared for *C* and *W* crystals in Figs. 6 and 7. In this calculation the AW amplitude is fixed at 100 nm . Other parameters are as indicated. These figures show that the properties of the AIR radiation depend also on the type of the material the undulator is made from. By varying the material one can achieve various radiation intensities at the same parameters of the AW.

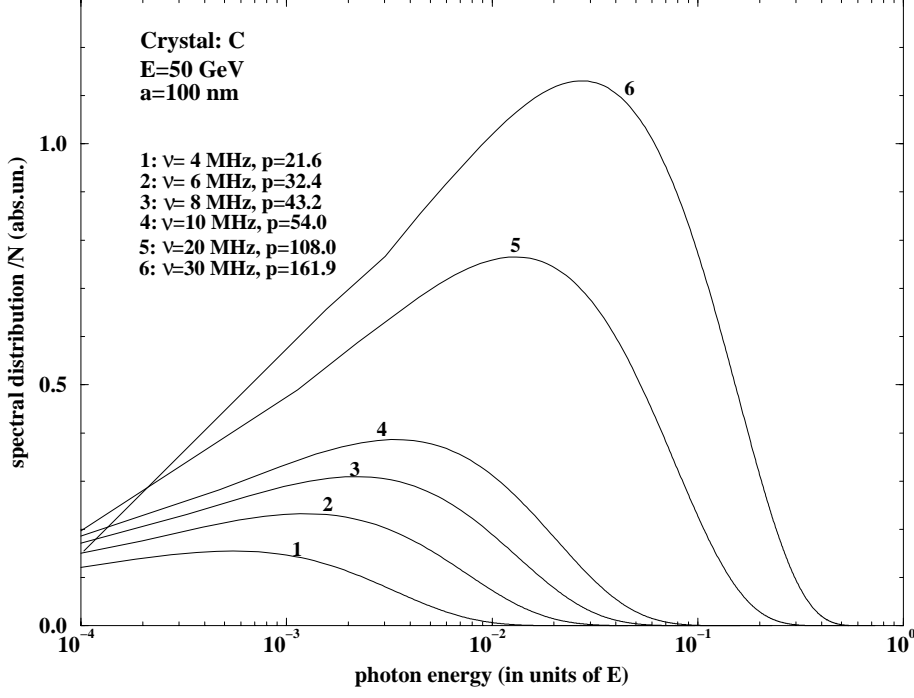
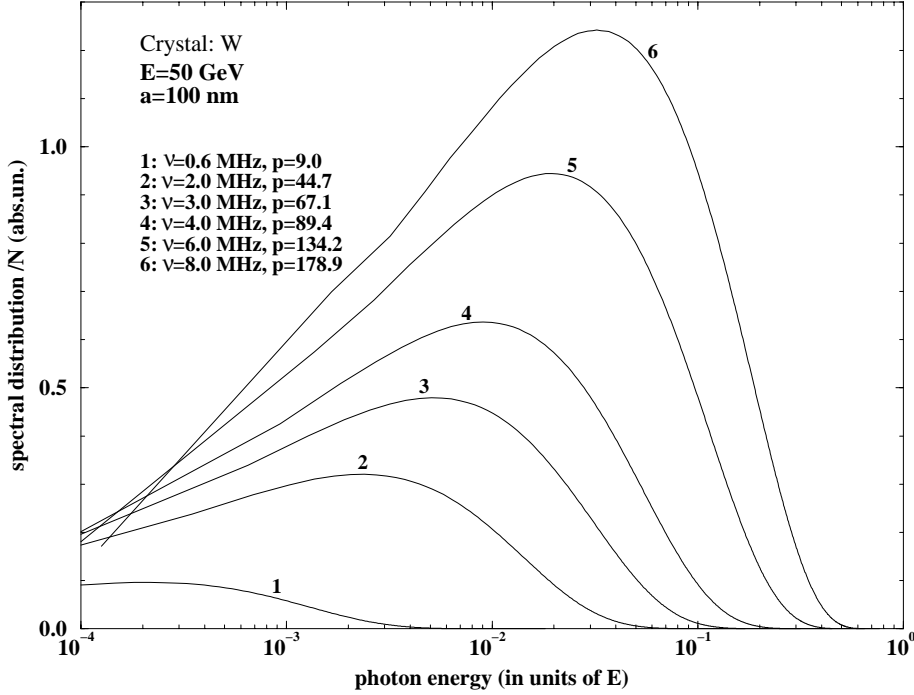


FIG. 6. The spectral intensity (per one period) of the AIR emitted by a 50 GeV positron in a carbon crystal calculated for the fixed AW amplitude (as indicated) and for various parameters ν and p (as indicated). See also explanations in the text.



Finally, let us estimate the stability of the suggested undulator. From general theory of undulators one can deduce [1] that the relative deviation of the undulator resonance frequency $\Delta\omega/\omega$ is proportional to the relative variation of the undulator parameters, in our case to $\Delta a/a$ and $\Delta\lambda/\lambda$. For the resonant undulator frequencies and the parameters considered above, the ratio $\Delta\omega/\omega \sim 0.1$. This means that the fluctuation of a and λ in the AW on the level of 10% or less does not influence much the stability of the suggested undulator even in the region of very high frequencies. Note that the relative variation of the AW amplitude during the time of flight of the positron through the crystal is much lower, $\Delta a/a \sim \nu L/c \sim 1/300$ at $\nu \sim 100\text{MHz}$ and $L \sim 1\text{cm}$.

Our investigation shows that the described phenomenon can be used for the construction of an undulator with variable parameters for the generation of high energy photons in a wide range. We have discussed the plane channeling of positrons combined with one of the most simple examples of AW. However, other cases of AW (longitudinal waves, spherical waves, non-monochromatic waves and various combinations thereof), interacting with the beam of the channeled particles (positrons, electrons, heavy ions) as well as the case of axial channeling of electrons are worthy to study. Another interesting question, being raised by our work, is the possibility of the stimulated photon emission in the undulator described above (free electron laser type). Such work is in progress.

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